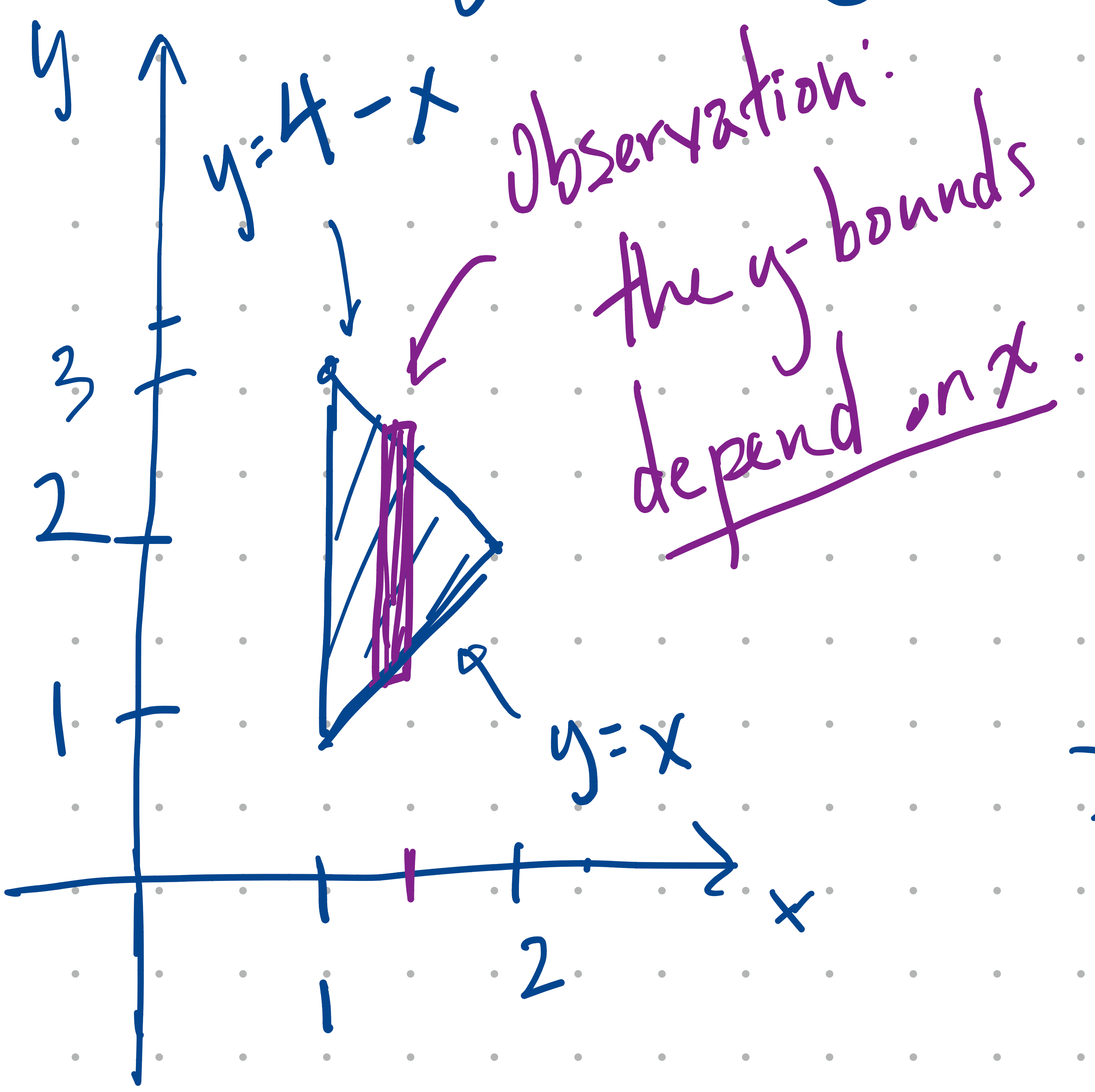
§15.2 OVERVIEW

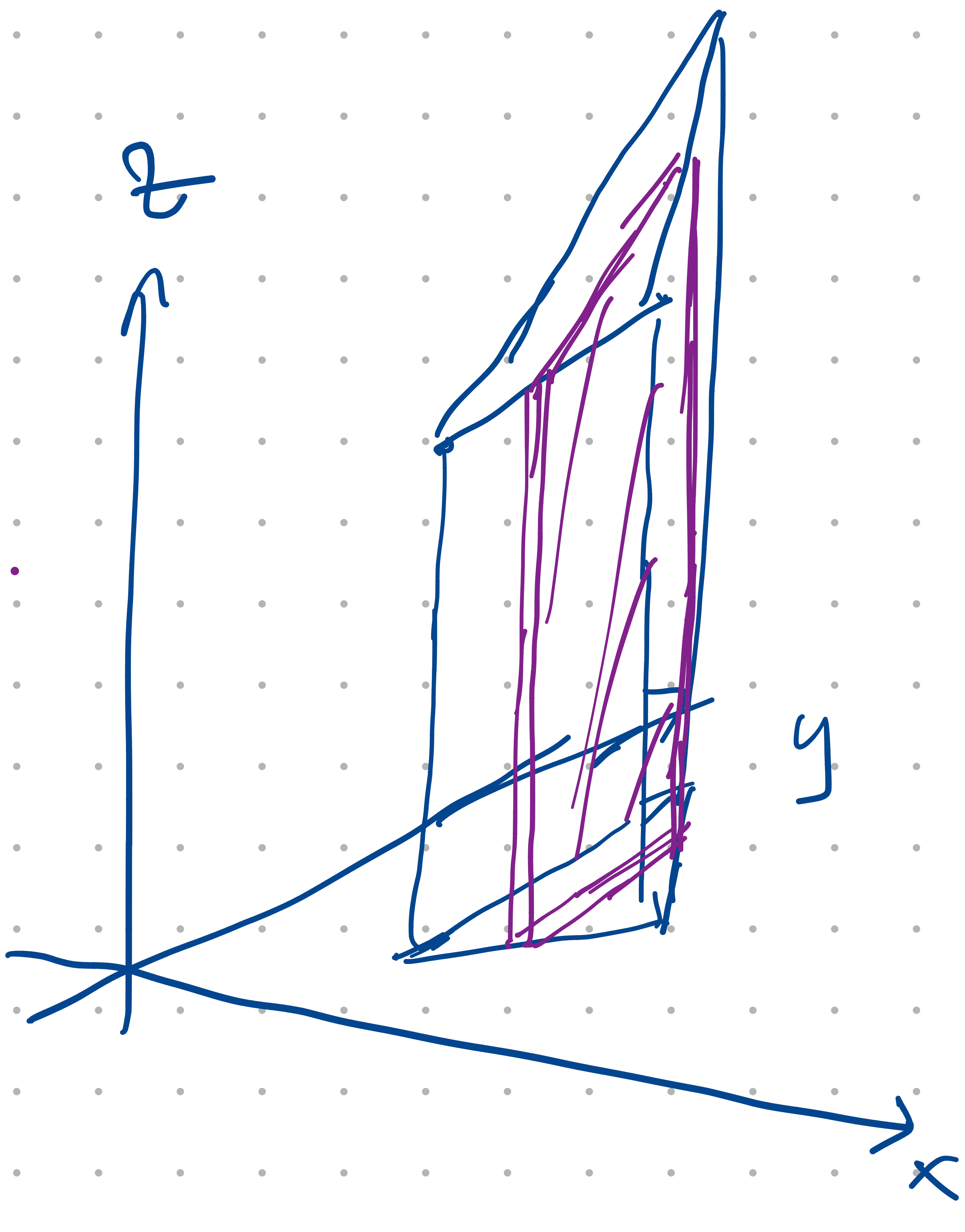
Last time, we studied integrals of the form $\int_a^b \int_c^d f(x,y)\,dy\,dx$, i.e.

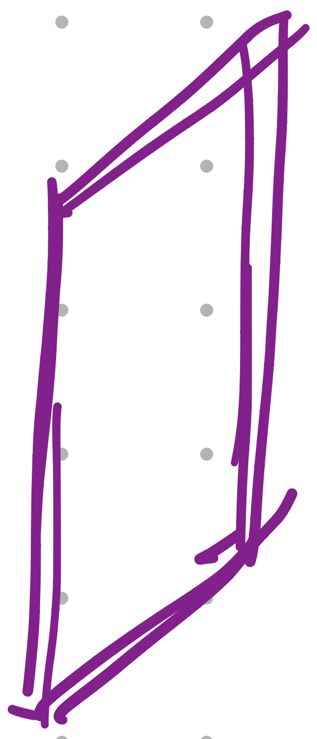
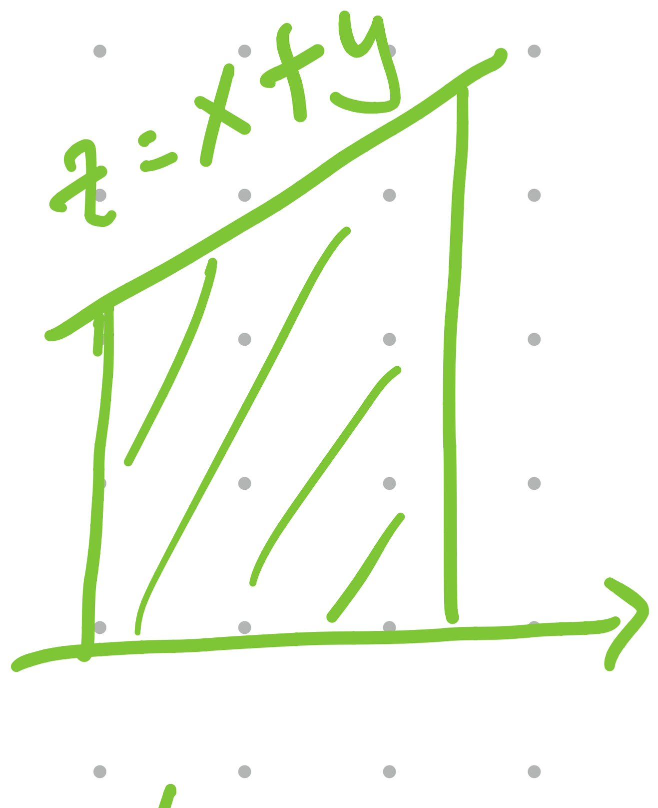
double integrals over rectangles.

Goal for today: drop this restriction.

ex) $f(x,y) = x+y$

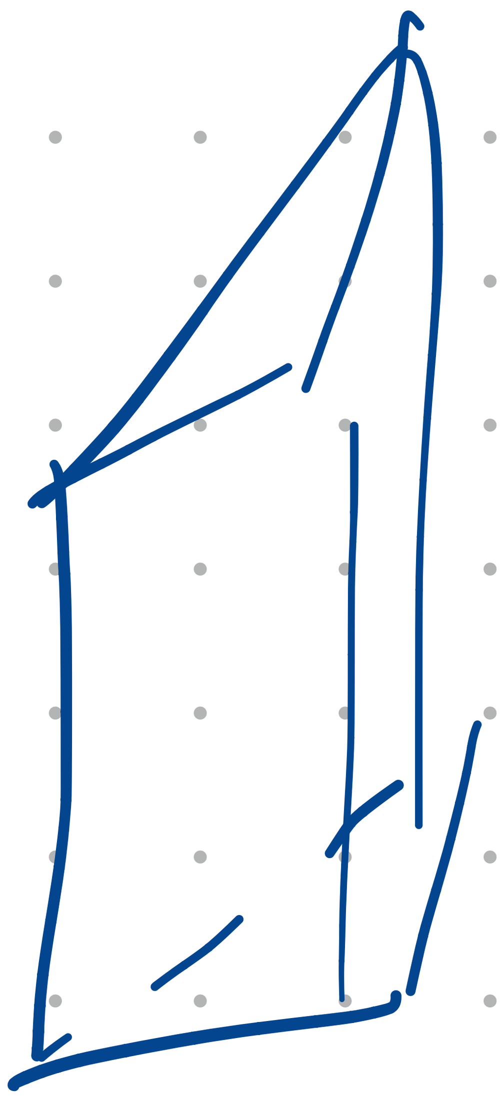




Volume of  = "() dx"

$$\parallel$$

$$\int_x^{4-x} (x+y)\,dy$$

Volume of  $= \int_1^2 \left(\int_x^{4-x} (x+y)\,dy \right) dx$

⚠ When doing integration order $dy\,dx$, the y-bounds (i.e. inner bounds) <u>may depend on x</u>. But the x-bounds (i.e. outer) are constant.

Let's compute:

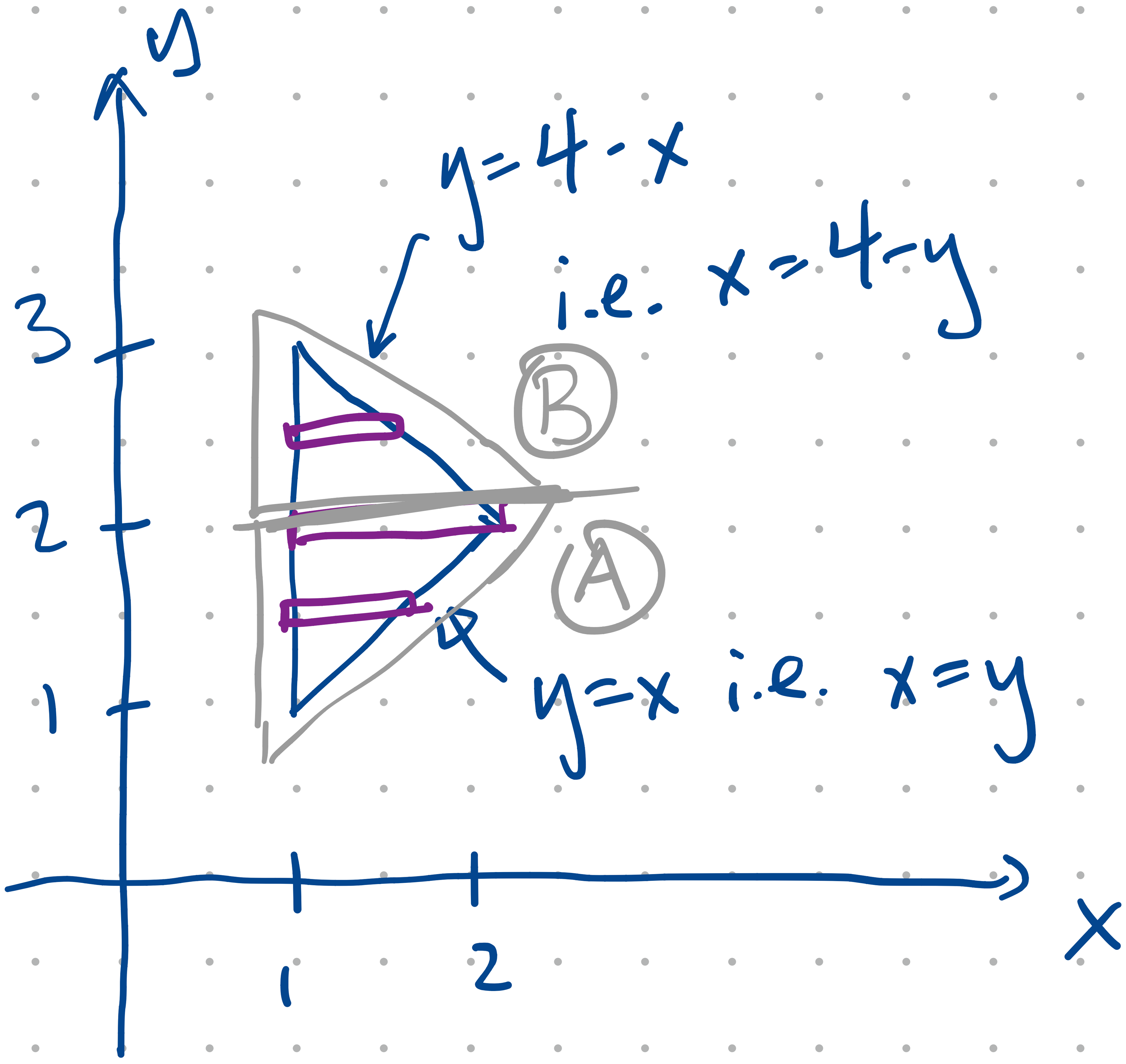
$$\int_1^2 \int_x^{4-x} (x+y)\,dy\,dx = \int_1^2 \left(xy + \frac{1}{2}y^2\right)\Bigg|_{y=x}^{y=4-x} dx$$

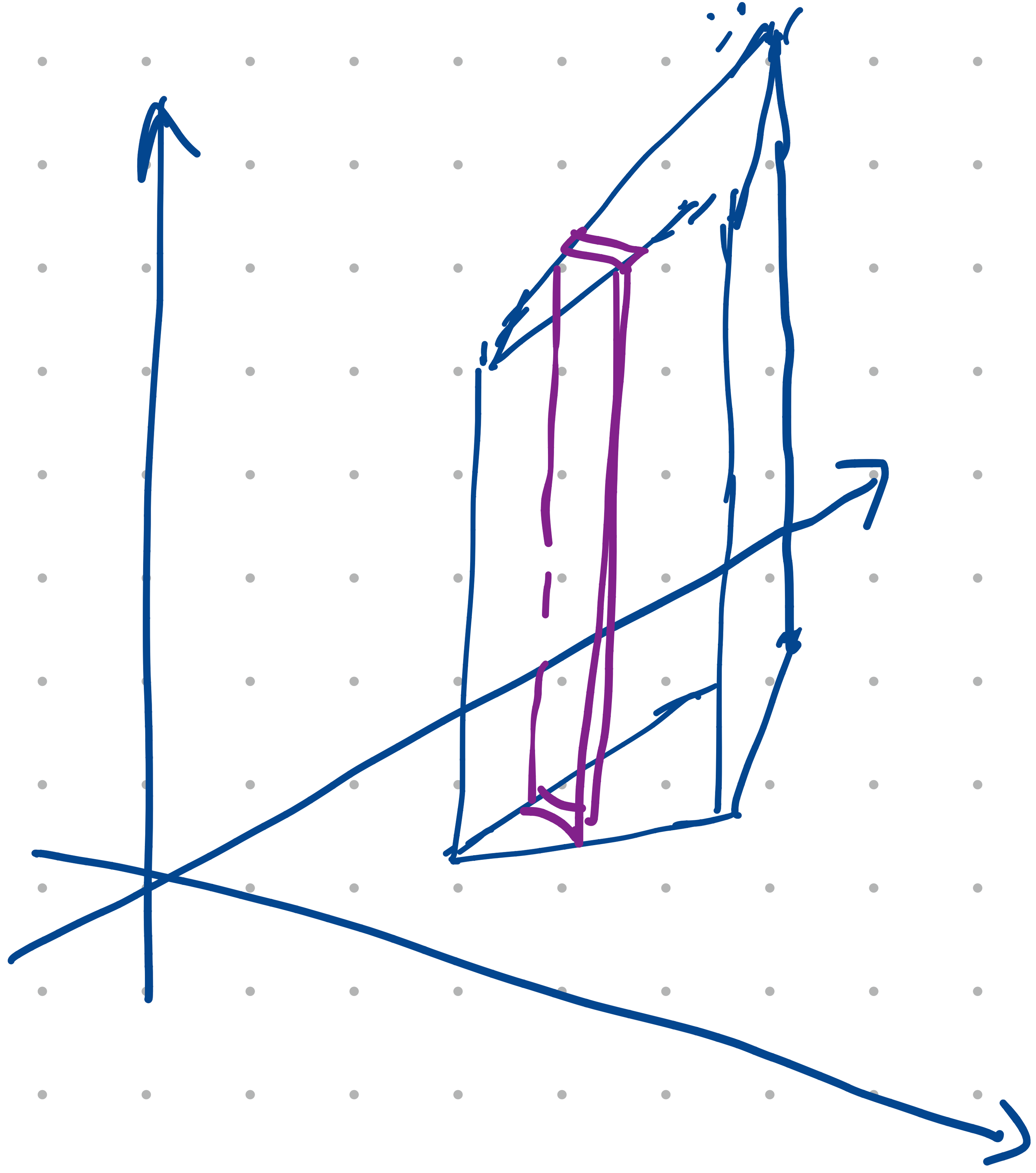
$$= \int_1^2 \left(x(4-x) + \frac{1}{2}(4-x)^2\right) - \left(x^2 + \frac{1}{2}x^2\right) dx$$

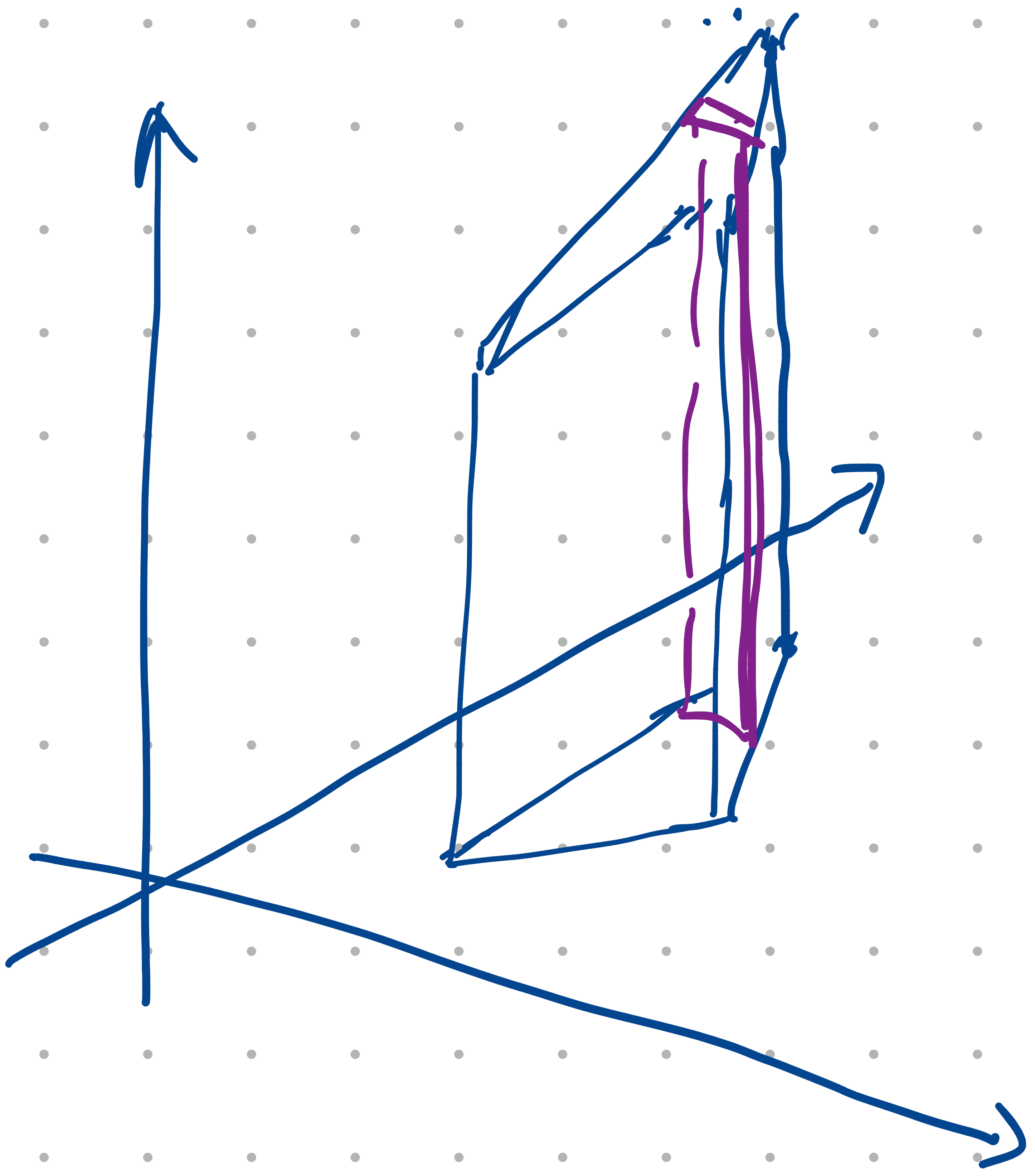
$$= \int_1^2 8 - 2x^2 \; dx \qquad \text{after simplifying}$$

$$= \left(8x - \frac{2}{3}x^3\right)\Bigg|_{x=1}^{2} = \left(16 - \frac{16}{3}\right) - \left(8 - \frac{2}{3}\right)$$

$$= \boxed{10/3}$$





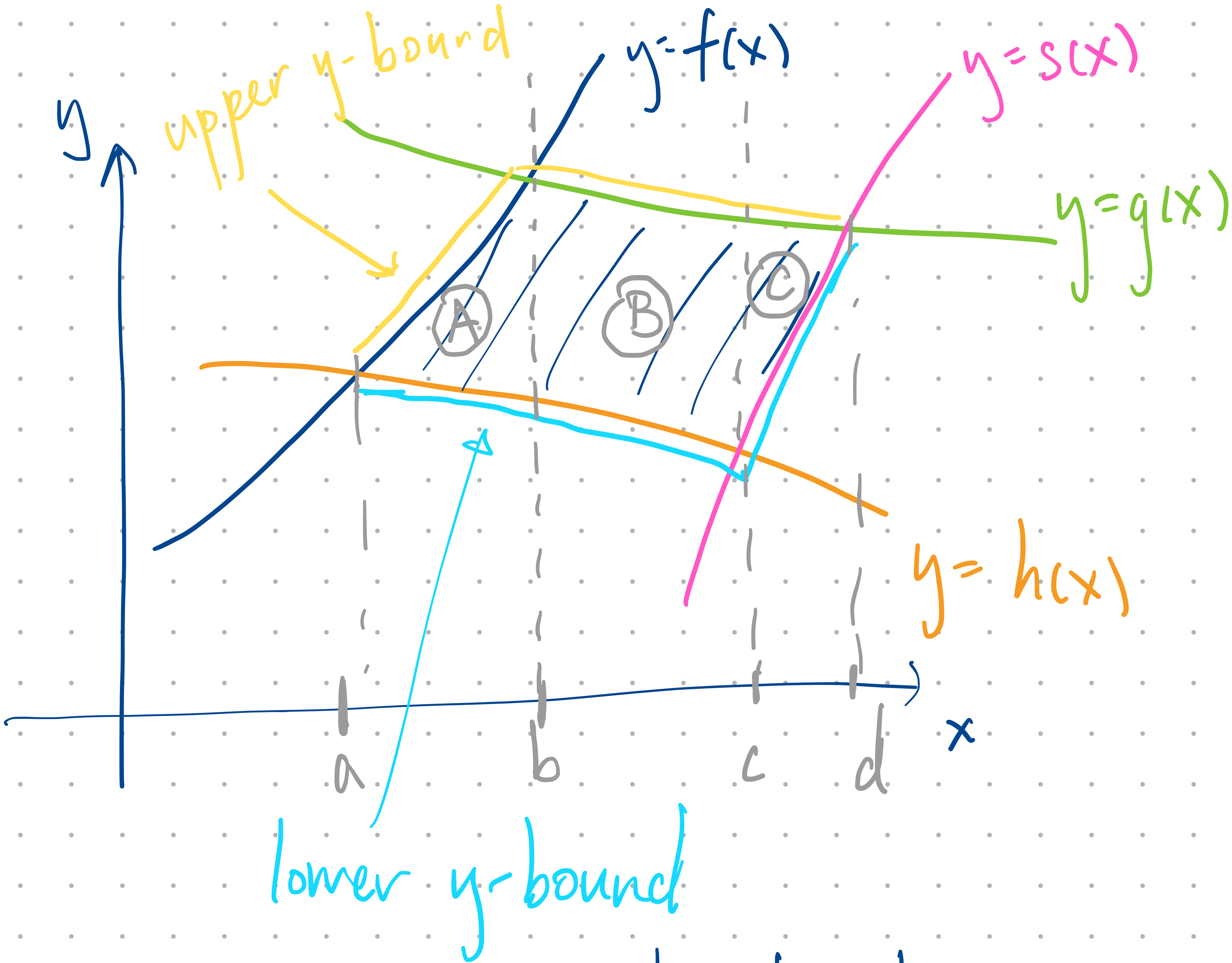


We need two integrals b/c the upper x-bound is a piecewise function of y.

$$\underbrace{\int_1^2 \int_1^y (x+y)\,dx\,dy}_{A} + \underbrace{\int_2^3 \int_1^{4-y} (x+y)\,dx\,dy}_{B}$$

If you were to compute this expression, you would end up with the same answer $10/3$.

⚠ If integrating $dx\,dy$, the x-bounds (inner) can depend on y. The y-bounds (outer) are constants.



Suppose we want to integrate $u(x,y)$ over the shaded region w/ $dy\,dx$ order:

$$\underbrace{\int_a^b \int_{h(x)}^{f(x)} u(x,y)\,dy\,dx}_{A} + \underbrace{\int_b^c \int_{h(x)}^{g(x)} u(x,y)\,dy\,dx}_{B} + \underbrace{\int_c^d \int_{s(x)}^{g(x)} u(x,y)\,dy\,dx}_{C}$$

Suppose we want to integrate $f(x, y)$ over the parallelogram bounded by the lines $y = x, y = 2x, y = x + 1, y = 2x - 1$. (Draw a picture.) If we use the integration order $dydx$, how many integrals do we have to write? (Try to actually write them.)

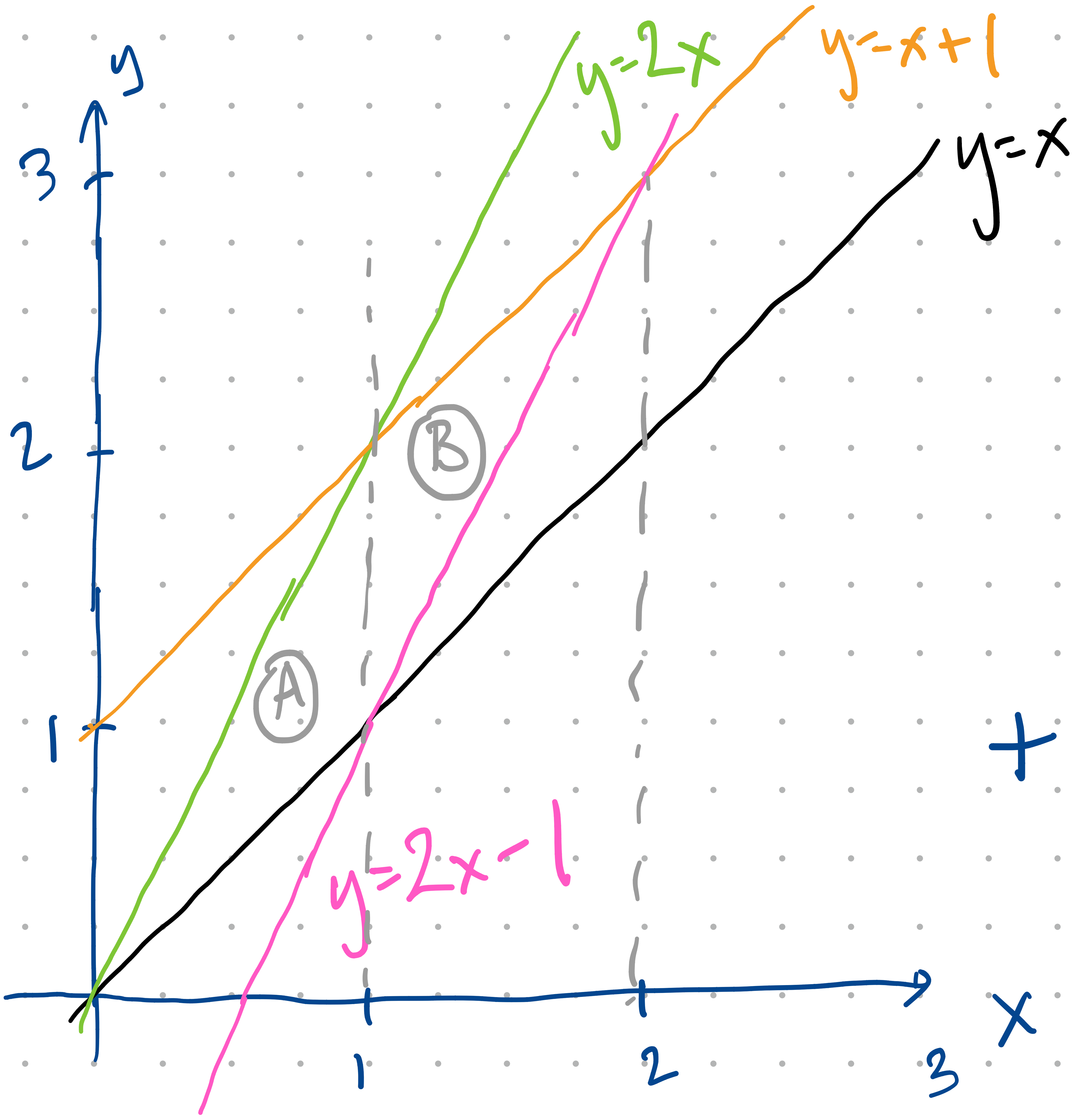
- 1
- 2
- 3
- More than 3

Total Results: 9

Suppose we want to integrate $f(x, y)$ over the parallelogram bounded by the lines $y = x, y = 2x, y = x + 1, y = 2x - 1$. (Draw a picture.) If we use the integration order $dxdy$, how many integrals do we have to write? (Try to actually write them.)

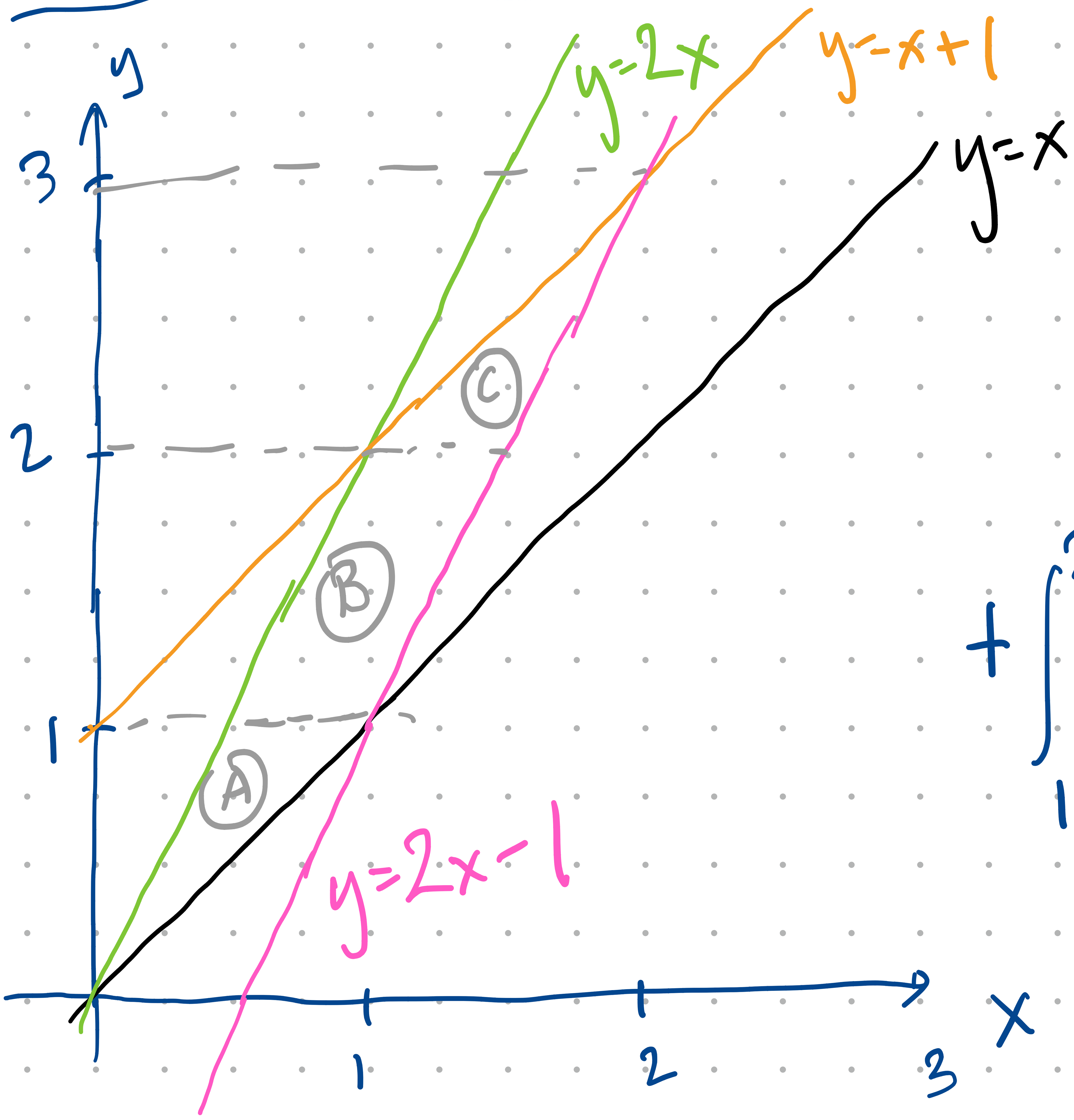
- 1
- 2
- 3
- More than 3

Total Results: 8



$$\int_0^1 \int_x^{2x} f(x,y)\,dy\,dx \quad \text{A}$$

$$+ \int_1^2 \int_{2x-1}^{x+1} f(x,y)\,dy\,dx \quad \text{B}$$



$$\int_0^1 \int_{y/2}^{y} f(x,y)\,dx\,dy \quad \text{A}$$

$$+ \int_1^2 \int_{y/2}^{\frac{y+1}{2}} f(x,y)\,dx\,dy \quad \text{B}$$

$$+ \int_2^3 \int_{y-1}^{\frac{y+1}{2}} f(x,y)\,dx\,dy \quad \text{C}$$